Fundamental sensitivity limit imposed by dark 1/f noise in the low optical signal detection regime

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Abstract: The impact of dark 1/f noise on fundamental signal sensitivity in direct low optical signal detection is an understudied issue. In this theoretical manuscript, we study the limitations of an idealized detector with a combination of white noise and 1/f noise, operating in detector dark noise limited mode. In contrast to white noise limited detection schemes, for which there is no fundamental minimum signal sensitivity limit, we find that the 1/f noise characteristics, including the noise exponent factor and the relative amplitudes of white and 1/f noise, set a fundamental limit on the minimum signal that such a detector can detect.

OCIS codes: (230.5160) Photodetectors; (040.3780) Low light level

References and links

1. S. O. Flyckt and C. Marmonier, *Photomultiplier tubes: Principles and applications* (Photonis, 2002).
2. M. C. Teich, K. Matsuo, and B. E. A. Saleh, "Excess noise factors for conventional and superlattice avalanche photodiodes and photomultiplier tubes," IEEE J. Quantum Electron. **22,** 1184-1193 (1986).
3. E. J. McDowell, X. Cui, Y. Yaqoob, and C. Yang, "A generalized noise variance analysis model and its application to the characterization of 1/f noise," Opt. Express **15,** (2007).
4. P. Dutta and P. M. Horn, "Low-frequency fluctuations in solids - 1/f noise," Rev. Mod. Phys. **53,** 497-516 (1981).
5. W. H. Press, "Flicker noises in astronomy and elsewhere," Comments Astrophys. **7,** 103-119 (1978).
6. M. B. Weissman, "1/f noise and other slow, nonexponential kinetics in condensed matter," Rev. Mod. Phys. **60,** 537-571 (1988).
7. E. Milotti, "1/f noise: A pedagogical review," invited talk to E-GLEA-2 (2001).
8. D. W. Allan, "Statistics of atomic frequency standards," Proc. Inst. Electr. Eng. **54,** 221-230 (1966).
9. J. A. Barnes, A. R. Chi, L. S. Cutler, D. J. Healey, D. B. Leeson, T. E. McGuniga, J. A. Mullen, W. L. Smith, R. L. Sydnor, R. F. C. Vessot, and G. M. R. Winkler, "Characterization of frequency stability," IEEE Trans. Instrum. Meas. **20,** 105-120 (1971).
10. I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Elsevier, 2007).

1. Introduction

The ability to detect very low amplitude optical signals is important and relevant in a variety of measurement scenarios. The detection of light from a distant star is an example. Additionally, many biomedical imaging techniques depend on measuring very weak optical signals, including second harmonic generation, Raman scattering, and single molecule fluorescence. For an ideal measurement system that contains only white noise sources, it is possible to measure any arbitrarily small signal by simply increasing the integration time of the detection system. However, this may not be possible in certain situations due to the presence of 1/f noise. In this manuscript, we will theoretically determine the detection limit imposed by 1/f noise, specifically dark 1/f noise, in direct low optical signal detection schemes.

In order to enable the detection of weak signals, it is common to use highly sensitive detectors such as photomultiplier tubes (PMTs) or avalanche photodiodes (APDs). PMTs utilize a combination of high gain, low noise, high frequency response, and large collection

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area [1] to achieve high sensitivity. APDs can be thought of as the semiconductor analog to a PMT.

Noise in optical detection is a combination of the intrinsic noise associated with a flow of photons, as well as noise associated with the detector. In the shot noise limit, noise related to the discrete arrival time of photons dominates all other noise processes. This limit represents an optical system functioning in an optimal manner, as the total noise of the detection process can never be reduced below the intrinsic shot noise level (without resorting to manipulation of the photon statistics). It is also possible for an optical system to operate such that detector noise is dominant. In broad terms, detector noise can be divided into two categories: dark noise and bright noise. Bright noise can arise as multiplicative noise caused by gain fluctuations and randomness in the carrier multiplication process of the PMT or APD [2], among other possible sources. In other words, if we are to put N photons on a detector, the count we receive will fluctuate around the mean value of εN (ε is the detector efficiency) due to randomness in the signal conversion process; this fluctuation comprises bright noise. Dark noise describes the random signal count from a detector that is blocked from receiving any optical signals. Among other causes, this can arise from thermally induced fluctuations and other additive noise sources in the detector. In the presence of a weak or absent input light field, bright noise can be neglected and dark noise dominates. As the focus of this study is centered on scenarios where the input light field is weak, we are primarily interested in the detector dark noise, specifically the dark 1/f noise.

Interestingly, the typical noise characteristics specified for high sensitivity optical detectors, including the noise equivalent power / bandwidth (NEP, NEB), reflect only the white noise portion of the dark noise. We note that such a characterization is incomplete, as the detector circuitry, among other potential sources, necessarily contributes dark 1/f noise. In the presence of dark 1/f noise, such devices will deviate from their predicted performance in which only dark white noise is accounted for. Our goal in this manuscript is to quantify this deviation. In particular, we will study the impact of dark 1/f noise on the fundamental sensitivity limit imposed by the detector that, to our knowledge, has never been studied or analyzed. This analysis is distinct from and complementary to our previous study [3] of the impact of 1/f noise in homodyne interferometric systems where 1) bright, rather than dark, detector noise was dominant, and 2) the noise analysis focused on the correct reception of a time varying signal trace rather than a confirmation of the existence of a signal source.

In this manuscript we first give a general description of 1/f noise, as well as several relevant studies of its effect in detection systems. We then define the specific problem that we are considering in this work. Next, we demonstrate the relevance of this analysis by experimentally showing that dark 1/f noise exists in practical detectors. The remainder of the manuscript is intended to be purely theoretical. We derive an expression for the SNR of the measurement of interest. We then show the results of our analysis and discuss the application of these results to experimental scenarios. We conclude by placing these results in context with our previous study [3], and provide a generalized guide for characterizing 1/f noise that is useful for a broad range of optical detection applications.

2. Background

1/f noise, alternately referred to as pink or flicker noise, can be found in a wide range of physical systems [4-6]. Generally, 1/f noise is represented by a power spectral density (PSD) that follows the form $1/f^{\alpha}$, where α commonly ranges from 0.5 to 1.5 [7]. The origins of 1/f noise sources are not well understood. In fact, a 1/f power spectrum can arise from very different time traces (sharp bursts versus slower baseline drifts of the system). The origins of these noise sources are not the focus of this manuscript. In the context of our analysis of detection sensitivity, it is sufficient to quantitatively characterize 1/f noise based on empirical data without seeking the exact nature of the noise source.

For white noise sources, we expect the deviation in the signal (or noise) count to scale as the square root of the signal (or noise) count. It is this deviation, rather than the noise count itself, that limits detector sensitivity. This point will be more explicitly clarified later, but an

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intuitive understanding of this issue is not hard to grasp through the following example. Suppose we have a unity efficiency detector that is known to have an average dark white noise count rate of x photon/s. This implies that if we want to detect the presence of a weak light source (photon rate of y photon/s) with this detector in a measurement made over T seconds, we must make sure that the total signal count (yT) exceeds the noise deviation term ($\sqrt{xT}$) rather than the total expected dark count (xT).

1/f noise differs from white noise in two distinct ways. First, 1/f noise differs in its dependence on the integration time of the detection system, which we will derive in this manuscript. Secondly, 1/f noise is dependent on an additional intrinsic factor, the noise exponent, α, which can vary from noise source to noise source.

Several published works have attempted to characterize the effects of 1/f noise on detection systems. Allan [8] has shown that there is a relationship between the PSD of the fluctuating phase of an atomic frequency standard and the variance of its frequency deviation. To derive this relationship, the variance of the frequency deviation is written in terms of the autocorrelation function of the phase, which can be related to the PSD of the phase using the Wiener-Khinchin Theorem. Allan's work illustrates a way to characterize the PSD of a random process through a statistically measurable quantity – the variance of a group of samples, where each sample is the time averaged value of the random variable over a given time period. The relationship between the variance and the PSD derived by Allan depends on the averaging time period, the dead time between samples, and the number of samples within the group. In a later work [9], the model was further generalized, and an analytical expression for the relationship was strictly proved.

More recently, we have described and verified a quantitative noise model to study the effect of 1/f noise in homodyne interferometers [3]. Our goal in doing so was essentially the opposite of Allan's. Our aim was to use the PSD of a noise source (which is measurable) to find the noise variance of a specific measurement we wish to make. In doing so we are then able to predict the signal-to-noise ratio (SNR) of the measurement. This quantitative noise model represents the noise in the system in a time domain format as a sum over all possible frequencies. Each frequency is represented by a sinusoidally varying term, weighted by the value of the PSD at that frequency. The phase of each term is random with respect to that of all other frequencies, essentially representing any and all possible time traces that can result from a superposition of those frequencies. Using this time domain representation, we were able to evaluate the noise variance that we would expect given the PSD of the noise source.

We further note that the 1/f noise studied in Ref. [3] was bright in nature rather than dark, as the reference beam of the homodyne interferometer was always incident on the detector. This strong light field precluded any significant dark noise contributions in that scenario.

3. Problem statement

Suppose we wish to confirm the existence of a weak light source, using a detector with dark noise power spectral density $S(f)$, determined a priori, and a mean dark noise count rate x_{noise}. For the sake of clarity, we choose to quantify our signal in terms of photon count rate (x) and photon counts (X). We can assign a signal count rate of x_{signal}, attributed to the presence of the weak light source; in the weak source regime, $x_{noise} >> x_{signal}$. If we are given a time frame of T to detect the presence of the weak light source, we can perform an experiment by first blocking the detector and measuring the dark noise count ($X_{noise}(\tau)$) for a time period of $\tau = T/2$, then exposing the detector to the weak light source and measuring the combination of dark noise and signal for an equivalent time period ($X_{signal+noise}(\tau=T/2)$). Our goal is to determine, based on the PSD of the noise, whether or not it is possible to discriminate the presence of x_{signal} from this measurement.

Experimentally, the power spectrum, $S(f)$, of the detector noise can be determined a priori by 1) acquiring a measurement trace from the detector in the dark (where the time duration of the trace is much longer than any experiments we wish to perform), 2) computing the autocorrelation function, and 3) finding the Fourier transform of the autocorrelation function. A realistic model for an optical detector involves modeling $S(f)$ as a sum of mutually

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independent white and 1/f noise. By relating the variance of a measurement, $X(\tau)$, to the power spectrum of the detector, $S(f)$, we can assess the impact of 1/f noise.

It is evident that our measurement scenario is quite different from that analyzed in our previous noise study (Ref. [3]) for a homodyne interferometer. In that study, a strict assumption was made regarding the relationship between the integration time (τ) and the total time frame of the experiment (T). This assumption, $\tau \ll T$, is relevant in certain scenarios; for example, the case in which a transmitter is transmitting a message of duration T, where the signal varies over each time step τ. (Fig. 1(a)). The experimental scenario examined in the current study is different in that our signal collection time frame is comparable to the total experimental time frame. A naïve extension of the result of Ref. [3] will not achieve the correct result for this particular detection scheme. Here, instead of receiving an amplitude modulated message, we simply wish to find the limit in which we can detect the presence of the transmitter in the first place. Experimentally, this corresponds to data obtained by alternate measurements of signal and noise (Fig. 1(b)), as we have described above.

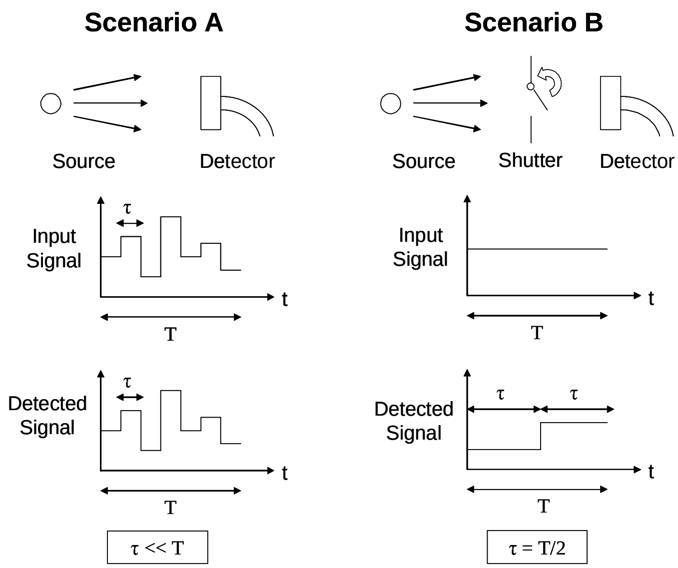

Fig. 1. A comparison between the measurement schemes in Ref. [3] (Scenario A) and the current work (Scenario B). In Scenario A, an amplitude modulated message is transmitted in steps of duration τ over a total time T. In this work, we simply wish to confirm the presence of the light source in an experiment where both signal and noise are measured for equivalent time periods.

4. Experimental verification

To confirm that dark 1/f noise does indeed exist in sensitive optical detectors, we measured the power spectral density of the dark count of an APD (Perkin-Elmer, SPCM-A2R15) over approximately 3 hours and plotted the result, averaged over 6 traces, in Fig. 2. We see that the noise spectrum can be well described as a combination of dark white and 1/f noise, with 1/f noise visible at and below frequencies in the mHz range.

Figure 2 shows that an α value of 1.6 was measured from the PSD of this particular APD. As our results in Ref. [3] showed that the noise exponent factor is highly device dependent for bright 1/f noise, it is possible that the α value for dark 1/f noise may vary significantly from device to device. The model that we present in this manuscript is applicable for any system with $\alpha>0$. We advise readers who intend to use our model in their respective applications to characterize their detectors via the above approach and calculate the corresponding α values.

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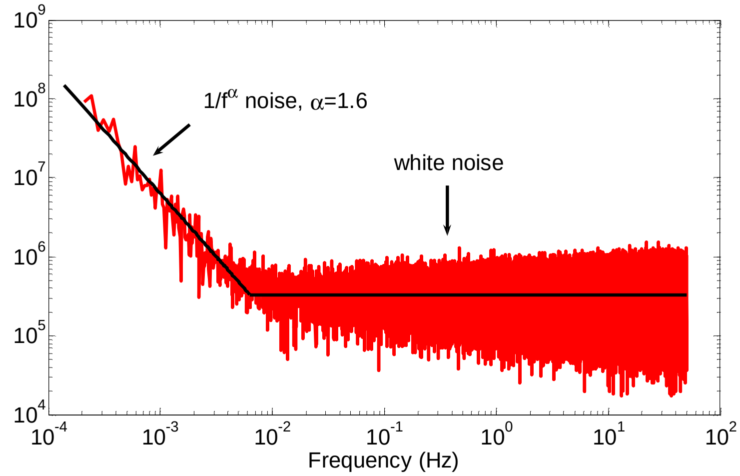

Fig. 2. Power spectral density of the dark noise count of a photon counting APD. 1/f noise is visible at and below frequencies in the mHz range. This averaged trace displays an α value of 1.6.

5. Theory

Given a measurement of the PSD of our detector dark noise, it is possible to derive the expected SNR of future measurements made with this detector. Our noisy signal can be described, in terms of photon count rate, as:

$$x(t) = x_{signal} + \Delta x(t), \tag{1}$$

where $\Delta x(t)$ represents the fluctuating noise. In our experiment, we make measurements of the signal and noise photon count over a time period, τ:

$$X_{noise}(\tau) = \int_0^{\tau} \Delta x(t)dt, \tag{2}$$

$$X_{signal+noise}(\tau) = \int_{\tau}^{2\tau} \left(x_{signal} + \Delta x(t)\right)dt. \tag{3}$$

In the context of this thought experiment, we wish to determine when the difference between the measurements described above gives a statistically significant result. Explicitly, we want to know when the difference between the mean signal and noise values is greater than the standard deviation of the measurement:

$$E\left(X_{signal+noise}(\tau) - X_{noise}(\tau)\right) \;>\; \sigma\left(X_{signal+noise}(\tau) - X_{noise}(\tau)\right). \tag{4}$$

The expected value of the left hand side of Eq. 4 is simply given by $x_{signal}\tau$. We can derive an expression for the variance of this measurement, the square of the right hand side of Eq. 4, as follows:

$$\sigma^2\left(X_{signal+noise} - X_{noise}\right) = E\left[\left(\int_{\tau}^{2\tau} \Delta x(t)dt - \int_0^{\tau} \Delta x(t)dt\right)^2\right]. \tag{5}$$

It can be seen from Eq. 5 that very low frequency components of Δx, which will essentially contribute the same number of photons to each integral, will cancel each other out. Thus, there is no dependence on the minimum measurable frequency, or alternately the total time frame of the experiment. We can combine the two integrals by making a change of variables:

$$\sigma^2\left(X_{signal+noise} - X_{noise}\right) = E\left[\left(\int_0^{\tau}(\Delta x(t+\tau) - \Delta x(t))dt\right)^2\right]. \tag{6}$$

This expression can be rewritten as:

$$\begin{aligned}
&\sigma^2\left(X_{signal+noise} - X_{noise}\right) \\
&= E\left[\int_0^{\tau}\int_0^{\tau}\Delta x(t_1+\tau)\Delta x(t_2+\tau)dt_1dt_2 + \int_0^{\tau}\int_0^{\tau}\Delta x(t_1)\Delta x(t_2)dt_1dt_2 - 2\int_0^{\tau}\int_0^{\tau}\Delta x(t_1+\tau)\Delta x(t_2)dt_1dt_2\right] \\
&= \int_0^{\tau}\int_0^{\tau}E(\Delta x(t_1+\tau)\Delta x(t_2+\tau))dt_1dt_2 + \int_0^{\tau}\int_0^{\tau}E(\Delta x(t_1)\Delta x(t_2))dt_1dt_2 - 2\int_0^{\tau}\int_0^{\tau}E(\Delta x(t_1+\tau)\Delta x(t_2))dt_1dt_2 \\
&= 2\int_0^{\tau}\int_0^{\tau}R(t_1-t_2)dt_1dt_2 - 2\int_0^{\tau}\int_0^{\tau}R(t_1-t_2+\tau)dt_1dt_2
\end{aligned} \tag{7}$$

where *R(t')* is the autocorrelation function and can be related to the PSD through the Wiener-Khinchin theorem as follows: $R(t') = \int_0^{\infty}S(f)\cos(2\pi f t')df$, for a single sided power spectrum, *S(f)*. For readers that are more familiar with a double sided power spectrum, we note that the single sided power spectrum simply folds the negative side of the even double sided power spectrum onto the positive frequency axis.

Eq. (7) can be evaluated as:

$$\begin{aligned}
&\sigma^2\left(X_{signal+noise} - X_{noise}\right) \\
&= 2\,\mathrm{Re}\left[\int_0^{\infty}\int_0^{\tau}\int_0^{\tau}S(f)e^{i2\pi f(t_1-t_2)}dfdt_1dt_2 - \int_0^{\infty}\int_0^{\tau}\int_0^{\tau}S(f)e^{i2\pi f(t_1-t_2+\tau)}dfdt_1dt_2\right] \\
&= 2\,\mathrm{Re}\left[\int_0^{\infty}\int_0^{\tau}\int_0^{\tau}S(f)e^{i2\pi f(t_1-t_2)}\left(1-e^{i2\pi f\tau}\right)dfdt_1dt_2\right] \\
&= 2\,\mathrm{Re}\left[\int_0^{\infty}S(f)\left(1-e^{i2\pi f\tau}\right)\left(\int_0^{\tau}e^{i2\pi f t_1}dt_1\right)\left(\int_0^{\tau}e^{-i2\pi f t_2}dt_2\right)df\right] \\
&= 2\,\mathrm{Re}\left[\int_0^{\infty}S(f)\left(1-e^{i2\pi f\tau}\right)\frac{1}{(2\pi f)^2}\left(e^{i2\pi f\tau}-1\right)\left(e^{-i2\pi f\tau}-1\right)df\right] \\
&= 2\,\mathrm{Re}\left[\int_0^{\infty}S(f)\left(1-e^{i2\pi f\tau}\right)\frac{1}{(\pi f)^2}\sin^2(\pi f\tau)df\right] \\
&= 2\,\mathrm{Re}\left[\int_0^{\infty}S(f)e^{i\pi f\tau}\left(e^{-i\pi f\tau}-e^{i\pi f\tau}\right)\frac{1}{(\pi f)^2}\sin^2(\pi f\tau)df\right] \\
&= 2\,\mathrm{Re}\left[\int_0^{\infty}S(f)e^{i\pi f\tau}(-2i)\frac{1}{(\pi f)^2}\sin^3(\pi f\tau)df\right]
\end{aligned}$$

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$$= 2\,\mathrm{Re}\left[\int_0^\infty S(f)\left(\cos(\pi f\tau) + i\sin(\pi f\tau)\right)(-2i)\frac{1}{(\pi f)^2}\sin^3(\pi f\tau)df\right]$$
$$= 4\int_0^\infty \frac{S(f)}{(\pi f)^2}\sin^4(\pi f\tau)df \tag{8}$$

From here, the power spectral density of the dominant noise source can be used to determine the expected noise variance, as well as the expected SNR of a measurement given knowledge of the signal amplitude. For dominant dark white noise, *S(f)* is a constant, A_{white}. The resulting noise variance and corresponding SNR are determined (using Eq. 3.822.12 and 3.828.13 of Ref [10]) to be:

$$\sigma_{white}^2(\tau) = \int_0^\infty \frac{4A_{white}}{\pi^2 f^2}\sin^4(\pi f\tau)df = A_{white}\tau\,, \tag{9}$$

$$SNR_{white} = \frac{x_{signal}\sqrt{\tau}}{\sqrt{A_{white}}}. \tag{10}$$

We can see that for a dark white noise limited signal, any arbitrarily small x_{signal} can be detected using a sufficiently long collection time, τ.

As we alluded to previously in this manuscript, the same is not true for the case of dominant dark 1/f noise. Here, we substitute a power spectrum of the form: $S(f) = A_{1/f} / f^{\alpha}$, and find a noise variance of the form:

$$\sigma_{1/f}^2(\tau) = \int_0^\infty \frac{4A_{1/f}}{\pi^2 f^{2+\alpha}}\sin^4(\pi f\tau)df\,. \tag{11}$$

This result can be reduced to the following form:

$$\sigma_{1/f}^2(\tau) = A_{1/f}C\tau^{1+\alpha}\,, \tag{12}$$

$$C = \left(2^{2+\alpha} - 2^{1+2\alpha}\right)\pi^{\alpha-1}\Gamma(-1-\alpha)\sin\left(\frac{\alpha\pi}{2}\right) \qquad \alpha > 0,\quad \alpha \notin Z\,,$$

$$SNR_{1/f} = \frac{x_{signal}\tau}{\sqrt{A_{1/f}C\tau^{1+\alpha}}} = \frac{x_{signal}\tau^{(1-\alpha)/2}}{\sqrt{A_{1/f}C}}\,. \tag{13}$$

Here, Γ represents the Gamma function. The derivation required for simplifying Eq. 11 makes use of known integral forms, including Eq. 3.756.4 and 3.756.9 in Ref. [10]. This solution holds for any non-integer value of α that is greater than zero. We can clearly see the dependence of the noise variance on the integration time, τ. Interestingly, we see from the SNR expression that for $\alpha>1$ we actually expect the SNR to decrease as a function of integration time.

Thus far, we have derived expressions for the noise variance given either white or 1/f noise. Since these two noise sources are independent of one another, we can describe the total noise variance as the sum of the individual variances, allowing us to examine the SNR that we might expect from a realistic optical system:

$$\sigma_{total}^2(\tau) = A_{white}\tau + A_{1/f}C\tau^{1+\alpha}\,, \tag{14}$$

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giving a combined SNR of:

$$SNR = \frac{x_{signal}\tau}{\sqrt{A_{white}\tau + A_{1/f}C\tau^{1+\alpha}}} = \frac{x_{signal}\sqrt{\tau}}{\sqrt{A_{white} + A_{1/f}C\tau^{\alpha}}}. \qquad (15)$$

This expression combines two competing terms: white noise dominated SNR which will improve with increasing integration time, and 1/f noise dominated SNR which will decrease (for α>1). The combination of these two noise sources suggests that the SNR will increase to a maximum value before beginning to decrease with integration time. More importantly, the presence of this maximal SNR value implies that there is a limit on the smallest x_{signal} that can be detected with such a system. The integration time at which the maximal SNR is achieved can be determined by solving for the time at which the derivative of the SNR expression is equal to zero. This results in an expression for τ of the following form:

$$\tau_{opt} = \left[\frac{A_{1/f}C}{A_{white}}(-1+2\alpha)\right]^{-1/\alpha}, \qquad (16)$$

where we can clearly see that the optimal integration time is a function of the relative amplitudes of dark 1/f and white noise in the system.

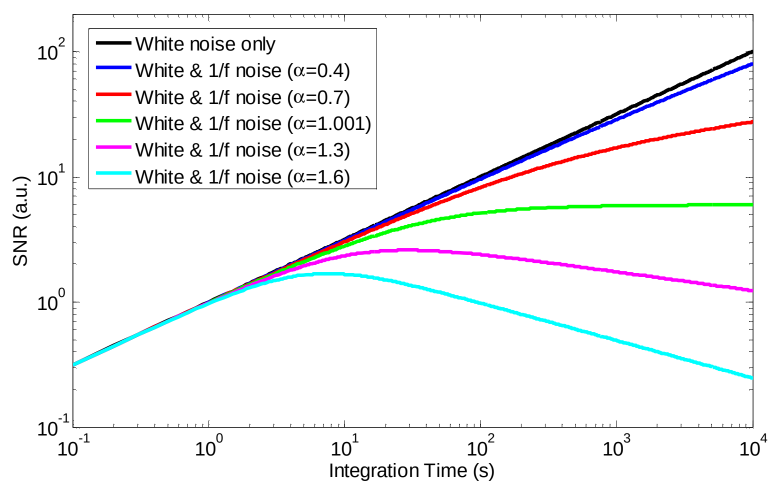

Fig. 3. SNR versus integration time for a combination of white noise and 1/f noise with α values ranging from 0.4 to 1.6 ($A_{1/f}/A_{white}$=0.01). The slope of the SNR trace decreases with increasing α. For $\alpha > 1$, the SNR reaches a peak value and begins to decrease with increasing integration time. The existence of a peak SNR value implies that there is a limit on the smallest signal that the system is capable of measuring.

6. Results

In the following paragraphs we will examine the behavior of the SNR expression in Eq. 15 as a function of α, as well as the relative amplitudes of white and 1/f noise. Figure 3 shows the SNR as a function of integration time for α values ranging from 0.4 to 1.6. We chose to fix the relative amplitudes of white and 1/f noise at $A_{1/f}/A_{white}$=0.01 for the purpose of this illustration. For α<1, the SNR steadily increases as a function of integration time, however at a slower slope as α nears 1. For α>1, the SNR curve reaches a maximum value and begins to decrease as a function of integration time.

The location of the maximal SNR value is dependent on the relative amplitudes of white and 1/f noise (Eq. 16). Figure 4 shows SNR traces for a fixed α value of 1.6. As $A_{1/f}/A_{white}$ is increased, we see the location of the maximal SNR (indicated by stars) moving toward shorter

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integration times. The dashed curve in Fig. 4 corresponds to an SNR trace that we might expect from the photon counting APD described above. The power spectrum in Fig. 2 was used to determine noise amplitude values ($A_{1/f}/A_{white}$=3.03x10^{-4}). This curve shows that the optimal integration time for the APD is approximately 50 s. Any further increase in integration time beyond this point will no longer improve the SNR.

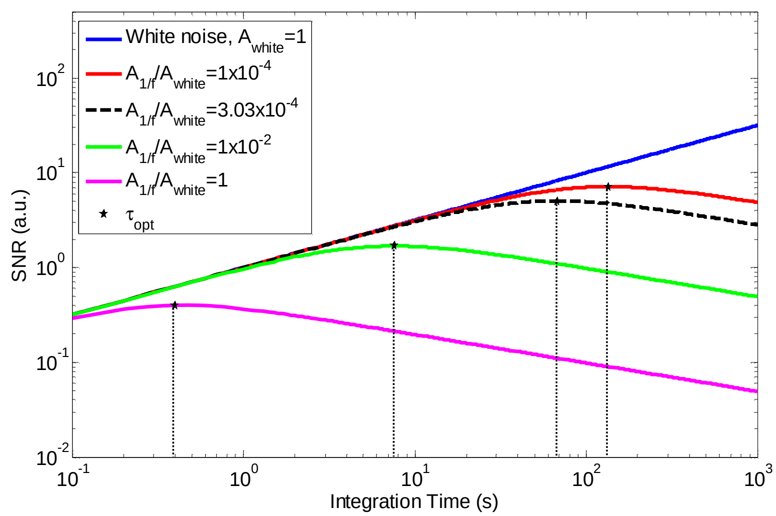

Fig. 4. The location of the peak SNR value is dependent on the relative amplitudes of white and 1/f noise. As $A_{1/f}/A_{white}$ is increased (for fixed α=1.6), the location of the maximum SNR, denoted by stars, moves towards shorter integration times. Curve fitting to the data shown in Fig. 1 we find $A_{1/f}/A_{white}$ = 3.03x10^{-4} for the photon counting APD described. The dashed curve shows an SNR trace corresponding to this value, with an optimal integration time of ~50 seconds.

7. Discussion

7.1 Summary of analysis

With the exception of Ref. [3], none of the studies mentioned above have applied their results to fundamental detection sensitivity in optical systems. The progression of our analysis (in terms of photon counts) and form of our solutions (in terms of SNR) are of direct and practical relevance for optical engineering. To our knowledge, this is the first study of the impact of 1/f noise in low optical signal direct detection schemes where dark noise dominates over other noise sources. We reinforce the fact that the thought experiment described here is different in implementation from the homodyne detection experiment in Ref. [3], as described in detail above. Although the two analyses give similar results for white noise dominated signals, the results in the presence of 1/f noise differ significantly. Ref. [3] showed that the SNR corresponding to Scenario A of Fig. 1 increased with increasing integration times before tapering to a constant value. In the current work, we have found that the SNR corresponding to Scenario B of Fig. 1 increases to a maximum before beginning to decrease with increasing integration times (for $\alpha > 1$).

The results of this analysis speak to the importance of careful photodetector selection. It is preferable to choose a detector with as small an α as possible. If α is less than 1, the detector is still capable of offering improved SNR with increasing integration time. If α is greater than one, there is a fundamental sensitivity limit associated with the detector that cannot be improved by increasing the signal integration time. Nevertheless, it is still desirable to aim for as small an α value as possible, as this will result in a broader peak in the SNR versus integration time curve. A broader peak implies that there is a broader range of integration times at which a high SNR can be obtained. In summary, the selection of a

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detector with a small α value is an important consideration when designing a weak signal detection system.

7.2 Application of analysis

The present work and our previous work (Ref. [3]) are complementary and widely applicable for noise characterization of detection schemes. The present work analyzes the fundamental detection limit in the context of dominant dark white and 1/f noise, but can also be used in scenarios where bright white and 1/f noise dominate (such as in homodyne or heterodyne detection). Our previous work (Ref. [3]), focused on the detection of a signal stream, was performed in the context where bright white and 1/f noise dominate, but, likewise, can be adapted for use in scenarios where dark white and 1/f noise dominate. Table 1 outlines the major results from the two analyses.

This sub-section aims to provide a recipe for choosing between the two analyses and appropriately applying them to specific detection scenarios. The steps are as follows:

Step 1: Determine if you are a) trying to confirm the existence of a light source or b) trying to receive a signal stream. Figure 1 can aid in your judgment.

Step 2: Obtain a detector time trace. This consists of acquiring a measurement trace from the detector with no useful optical signal incident on the detector. If you expect your

Table 1. A comparison of the important equations in both Ref. [3] (Scenario A) and the current study (Scenario B).

	Scenario A	**Scenario B**
Measurement	X_{signal}	$X_{signal+noise} - X_{noise}$
White Noise Variance	$\sigma^2_{white}(\tau) = \frac{1}{2} A_{white}\tau$	$\sigma^2_{white}(\tau) = A_{white}\tau$
1/f Noise Variance	$\sigma^2_{1/f}(\tau) \approx A_{1/f} B\tau^{\alpha+1}$ for $\alpha < 1,\ \ \alpha \notin Z$ $\sigma^2_{1/f}(\tau,T) \approx A_{1/f}\left[\frac{\tau^2 T^{\alpha-1}}{\alpha-1}\right]$ for $\alpha > 1,\ \ \alpha \notin Z$	$\sigma^2_{1/f}(\tau) = A_{1/f} C\tau^{1+\alpha}$ for $\alpha > 0,\ \ \alpha \notin Z$
Combined SNR	$SNR(\tau) \approx \frac{x_{signal}\sqrt{\tau}}{\sqrt{A_{white} + A_{1/f} B\tau^{\alpha}}}$ for $\alpha < 1,\ \ \alpha \notin Z$ $SNR(\tau,T) \approx \frac{x_{signal}\sqrt{\tau}}{\sqrt{A_{white} + \frac{A_{1/f}\tau T^{\alpha-1}}{\alpha-1}}}$ for $\alpha > 1,\ \ \alpha \notin Z$	$SNR = \frac{x_{signal}\sqrt{\tau}}{\sqrt{A_{white} + A_{1/f} C\tau^{\alpha}}}$ for $\alpha > 0,\ \ \alpha \notin Z$

The white noise variance is almost identical, regardless of the measurement scheme employed. In contrast, the 1/f noise variance differs in both its dependence on the integration time (τ), as well as its dependence on the total time frame of the experiment (in Scenario A only). The constant of proportionality, B is given by: $(2\pi)^{\alpha}/(2\alpha(\alpha+1)\Gamma(\alpha)\cos(\alpha\pi/2))$, and C can be found in Eq. 12 above.

experiment to be dominated by dark noise, this implies blocking all light from reaching the detector. If you expect your experiment to be dominated by bright noise, this implies only permitting background light power to reach the detector (such as the reference power in a heterodyne or homodyne interferometric system). The time duration for the trace should be much longer than the time frame of any experiments you wish to perform with the detector.

Step 3: Calculate the noise PSD by computing the autocorrelation function of the time trace, then finding the Fourier transform of the autocorrelation function.

Step 4: Extract the white and 1/f noise components from the noise PSD curve. The amplitudes of the two noise terms, as well as the α value of the 1/f noise term, should be determined.

Step 5: Compute the noise variances. For scenario A, you need the values calculated in Step 4, the detection integration time associated with each signal time step, τ, and the time frame of the experiment, T. Use Eq. 9 and 10 of Ref. [3]. For scenario B, you need the values calculated in Step 4 and the detection integration time, τ. Use Eq. 9 and 11 of this work.

Step 6: Calculate the associated SNR value. For scenario A, use Eq. 14 of Ref. [3]. For scenario B, use Eq. 15 of this work.

8. Conclusion

In conclusion, we have shown that there exists a theoretical fundamental sensitivity limit due to the presence of 1/f noise in low signal optical detection. We derive the signal to noise ratio that corresponds to a simple thought experiment, designed to detect the presence of a small signal buried in detector noise. For white noise dominated signals, our results are similar to those in Ref. [3]. However, they are quite different for the case in which 1/f noise dominates. This subtle point is particularly important and relevant to weak signal detection schemes, as the type of detection scheme involved can lead to different SNR characteristics. For the detection scenario discussed in this paper, our results show that for a combination of white noise and 1/f noise the SNR is continually increased with increasing integration times for $\alpha < 1$. However, for $\alpha > 1$, the SNR peaks and begins to decrease with integration time. This result implies a fundamental limit on the sensitivity of detection systems that operate in the presence of 1/f noise ($\alpha > 1$). Depending on the strength of the small signal and the 1/f noise characteristics, the optimal SNR may not be sufficient to enable weak signal detection regardless of the integration time involved.

On an intuitive level, the results make good sense, as we can expect to observe strong 1/f noise contributions corresponding to low frequencies when collecting signals over relatively long time scales. The lower the frequency of a particular 1/f noise component, the wider the integration window needs to be to observe its net effect. The overall behavior of the SNR will degrade as a function of the integration time when the linear signal strength increase (with respect to the integration time) is unable to compensate for the increased noise associated with the stronger low 1/f noise frequency components for ($\alpha > 1$). Purely white noise dominated systems do not face this issue because the linear increase in signal strength is always more rapid than the increase in white noise.

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