2π ambiguity-free optical distance measurement with subnanometer precision with a novel phase-crossing low-coherence interferometer

Changhuei Yang, Adam Wax, Ramachandra R. Dasari, and Michael S. Feld

George Harrison Spectroscopy Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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We report a highly accurate phase-based technique for measuring arbitrarily long optical distance with subnanometer precision. The method employs a Michelson interferometer with a pair of harmonically related light sources, one cw and the other low coherence. By slightly detuning (~2 nm) the center wavelength of the low-coherence source between scans of the target sample, we can use the phase relationship between the heterodyne signals of the cw and the low-coherence light to measure the separation between reflecting interfaces with subnanometer precision. As this technique is completely free of 2π ambiguity, an issue that plagues most phase-based techniques, it can be used to measure arbitrarily long optical distances without loss of precision. We demonstrate one application of this technique, the high-precision determination of the differential refractive index. © 2002 Optical Society of America

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Phase-based optical interferometric techniques are widely employed in optical distance measurements in which subwavelength distance sensitivity is required. However, most such techniques are limited by an issue that is widely known in the field as 2π ambiguity (or integer ambiguity),[1] the difficulty in telling the interference fringes of an axial scan apart from each other. As a result, these techniques are unable to determine optical distance absolutely. Therefore, most such techniques are used in applications, such as texture evaluating of continuous surfaces[2,3] or detecting time-dependent distance changes,[4] in which phase unwrapping is possible through comparison of phases between adjacent points or over small time increments. A recent novel phase technique overcomes the 2π ambiguity by introducing a dispersion imbalance in the interferometer; the method is able to measure the relative height difference of two adjacent points on a surface with great precision.[5]

Interferometric optical distance measuring systems employing readily available low-coherence light sources have achieved resolution of the order of tens of wavelengths.[6] Although such techniques are relatively insensitive, they do not have to contend with the 2π ambiguity issue. In this Letter, we present a novel low-coherence interferometry scheme that uses phase to measure arbitrarily long optical distances with subnanometer precision. This scheme uses a low-coherence phase-crossing technique to determine the integer number of interference fringes and additional phase information from the measurement to obtain the fractional fringe accurately. In addition, the scheme provides depth resolution and can be used for tomographic profiling. As the method can measure long optical distances with unprecedented precision, it can be used to determine the refractive indices of materials very accurately. We demonstrate this application for several materials.

The experimental setup employs a modified Michelson interferometer (Fig. 1). The input light is a two-color composite beam composed of 150-fs mode-locked light from a Ti:sapphire laser at 775.0 nm and cw 1550.0-nm light from a semiconductor laser. This method evaluates optical distances in terms of the cw wavelength. The composite beam is divided in two at the beam splitter. One part (the signal) is incident on the target, and the other is incident on a reference mirror moving at 0.5 mm/s, which induces a Doppler shift in the reference beam. The backreflected beams are recombined at the beam splitter, separated into their wavelength components by means of a dichroic mirror, and measured separately with photodetectors. The resulting heterodyne signals at their respective Doppler-shifted frequencies are bandpassed around their respective center heterodyne frequencies and Hilbert transformed for extraction of the corresponding phases of the heterodyne signals, Ψ_{cw} and Ψ_{LC}.[4] (The subscripts cw and LC denote the 1550.0-nm cw and 775.0-nm low-coherence wavelength components, respectively).

The principle of the technique can be illustrated through the simple case of a sample of thickness L

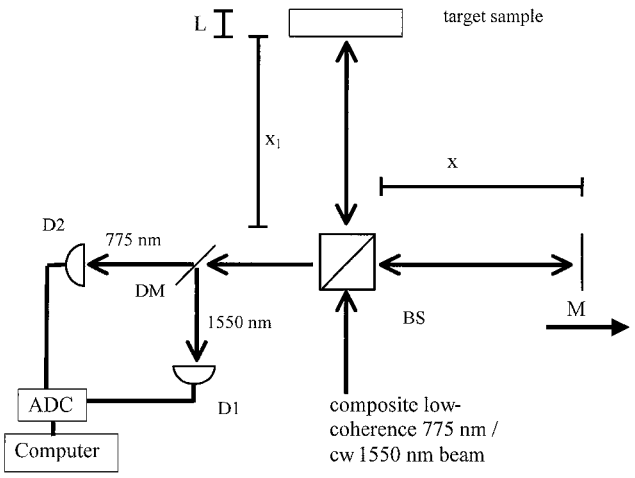

Fig. 1. Experimental setup: M, reference mirror; BS, beam splitter; D1, D2, photodetectors; DM, 775-nm/1550-nm dichroic mirror. ADC, analog–digital converter.

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and refractive index $n_{775\,\mathrm{nm}}$ at a wavelength of 775 nm. The two interfaces of the sample are at optical distances x_1 and x_2 ($=x_1 + n_{775\,\mathrm{nm}}L$) from the beam splitter, respectively. Note that the method works only if the optical distance separation is greater than the coherence length of the low-coherence light source. Otherwise, the heterodyne phase signals associated with the interfaces will merge, resulting in inaccurate interface localization. As the reference mirror is scanned, the phase of the low-coherence heterodyne signal is given by

$$\begin{aligned}\psi_{\mathrm{LC}}(x) &= \mathrm{mod}_{2\pi}\{\arg(R_{\mathrm{LC},1}\exp[i2k_{\mathrm{LC}}(x - x_1)] \\ &\quad \times \exp\{-[2a(x - x_1)]^2\} + R_{\mathrm{LC},2} \\ &\quad \times \exp[i2k_{\mathrm{LC}}(x - x_2)]\exp\{-[2a(x - x_2)]^2\})\} \\ &\approx h_c(x - x_1)\mathrm{mod}_{2\pi}[2k_{\mathrm{LC}}(x - x_1)] \\ &\quad + h_c(x - x_2)\mathrm{mod}_{2\pi}[2k_{\mathrm{LC}}(x - x_2)], \qquad (1)\end{aligned}$$

where $R_{\mathrm{LC},j}$ is the reflectivity of interface j at the low-coherence wavelength, k is the optical wave number, $a = 4\ln(2)/l_c$, l_c is the coherence length, x is the distance of the reference mirror from the beam splitter, and $h_c(x)$ is a piecewise continuous function with a value of 1 for $|x| < 2l_c$ and 0 otherwise. The factors of 2 in the exponents are due to the effective doubling of optical paths in the backreflection geometry. Equation (1) reflects the fact that phase cannot be measured far beyond the coherence envelopes, because of noise.

The phase of the cw heterodyne signal is given by

$$\begin{aligned}\psi_{\mathrm{cw}}(x) &= \mathrm{mod}_{2\pi}\{\arg(R_{\mathrm{cw},1}\exp[i2k_{\mathrm{cw}}(x - x_1)] \\ &\quad + R_{\mathrm{cw},2}\exp\{i2k_{\mathrm{cw}}[x - (x_1 + n_{1550\,\mathrm{nm}}L)]\}\})\} \\ &= \mathrm{mod}_{2\pi}(\arg\{\overline{R}\,\exp[i2k_{\mathrm{cw}}(x - \overline{x})]\}) \\ &= \mathrm{mod}_{2\pi}[2k_{\mathrm{cw}}(x - \overline{x})], \qquad (2)\end{aligned}$$

where $R_{\mathrm{cw},j}$ is the reflectivity of interface j at the cw wavelength, $n_{1550\,\mathrm{nm}}$ is the sample's refractive index, and $\overline{R}$ and $\overline{x}$ are the effective average reflectivity and distance from the beam splitter, respectively. If we choose the center wavelengths of the two light sources such that

$$k_{\mathrm{LC}} = 2k_{\mathrm{cw}} + \Delta, \qquad (3)$$

where Δ is a small intentionally added shift, then we can obtain a difference phase, ψ_D, of the form

$$\begin{aligned}\psi_D(x) &= \psi_{\mathrm{LC}}(x) - 2\psi_{\mathrm{cw}}(x) \\ &= h_c(x - x_1)\mathrm{mod}_{2\pi}[4k_{\mathrm{cw}}(\overline{x} - x_1) + 2\Delta(x - x_1)] \\ &\quad + h_c(x - x_2)\mathrm{mod}_{2\pi}[4k_{\mathrm{cw}}(\overline{x} - x_2) + 2\Delta(x - x_2)]. \qquad (4)\end{aligned}$$

Equation (4) provides both the approximate number of fringes in the interval $(x_2 - x_1)$ and the fractional fringe, which provides subwavelength precision.

As we vary Δ by a small amount (corresponding to a wavelength shift of 1–2 nm), the slope of $\psi_D(x)$ will pivot around the points where $x = x_1$ and $x = x_2$. In other words, the phase scans at different values of Δ will cross at those points. We can find the optical distance from x_1 to x_2 by counting the fringes that $\psi_{\mathrm{cw}}(x)$ goes through between the two phase-crossing points. We denote twice the quantity thus found as S_{fringe} (note that this is not an integer value), corresponding to the number of fringes at the low-coherence wavelength.

We can use the phase-shift information to localize the interface separation further. Specifically, the difference between the phase shifts at $x = x_1$ and $x = x_2$ is

$$\begin{aligned}S_{\mathrm{phase}} &= \frac{\mathrm{mod}_{2\pi}[\psi_D(x = x_1) - \psi_D(x = x_2)]}{2\pi} \\ &= \frac{\mathrm{mod}_{2\pi}[4k_{\mathrm{cw}}(x_2 - x_1)]}{2\pi}. \qquad (5)\end{aligned}$$

S_{phase} (5) measures the fractional fringe with great sensitivity.

The absolute optical separation $(x_2 - x_1)$ can be determined with great precision from S_{fringe} and S_{phase} through the following equation:

$$\begin{aligned}(x_2 - x_1)_{\mathrm{measured}} &= (n_{775\,\mathrm{nm}}L)_{\mathrm{measured}} \\ &= \frac{\lambda_{\mathrm{cw}}}{4}\left\{\left[\mathrm{int}(S_{\mathrm{fringe}}) + U\left(\Delta S - \frac{1}{2}\right)\right.\right. \\ &\quad \left.\left. - U\left(-\Delta S - \frac{1}{2}\right)\right] + S_{\mathrm{phase}}\right\}, \qquad (6)\end{aligned}$$

where $\Delta S = \mathrm{res}(S_{\mathrm{fringe}}) - S_{\mathrm{phase}}$ and $U()$ is a unit step function. Here, int() and res() denote the integer and fractional parts of the argument, respectively. The first term localizes the optical distance to the correct integer number of fringes by minimizing the error between S_{phase} and the fractional part of S_{fringe}. The error of an optical separation determination is limited only by the measurement error of S_{phase}. In our experiment, such error translates to an error in $(n_{775\,\mathrm{nm}}L)_{\mathrm{measured}}$ of ~0.5 nm. The measurement error of S_{fringe} need only be smaller than half a fringe

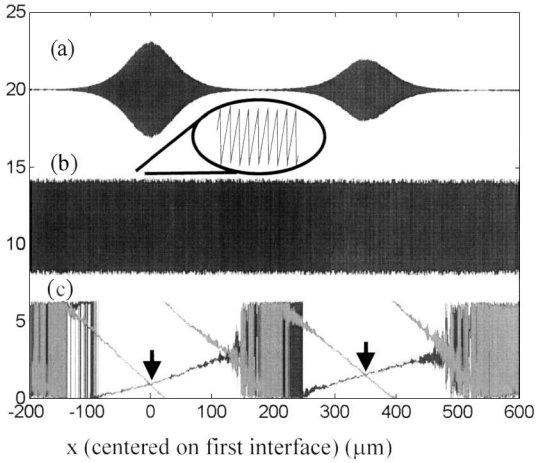

Fig. 2. Typical scan of a sample with two interfaces: (a) low-coherence heterodyne signal, (b) $\psi_{\mathrm{cw}}(x)$ trace. The magnified view in (b) shows the phase fringes. Each fringe corresponds to an optical distance of λ_{cw}. (c) Traces of $\psi_D(x)$ at two different values of Δ. The arrows indicate the phase-crossing points. In (b) and (c), the vertical axis is in radians.

Table 1. Measurments of $(n_{775\,\mathrm{nm}}L)$ of a Piece of Quartz Coverslip

Set	$\frac{\lambda_{cw}}{4} S_{fringe}$ (μm)	$\frac{\lambda_{cw}}{4} S_{phase}$ (μm)	$(n_{775\,\mathrm{nm}}L)_{measured}$ (μm)
1	350.86 ± 0.17	0.3496 ± 0.0004	351.0371 ± 0.0004
2	351.08 ± 0.17	0.3497 ± 0.0004	351.0372 ± 0.0004
3	351.15 ± 0.16	0.3502 ± 0.0004	351.0377 ± 0.0004
4	351.04 ± 0.18	0.3498 ± 0.0004	351.0373 ± 0.0004
Average			351.0373 ± 0.0004

Table 2. Measurements of $n_{775\,\mathrm{nm}}/n_{1550\,\mathrm{nm}}$ for Different Materials

Material	$n_{775\,\mathrm{nm}}/n_{1550\,\mathrm{nm}}$
Quartz	1.002742 ± 0.000003
Glass (German borosilicate)	1.008755 ± 0.000005
Acrylic plastic	1.061448 ± 0.000005

so that the correct interference fringe can be established; having satisfied this criterion, the measurement error of S_{phase} does not enter into the error of $(n_{775\,\mathrm{nm}}L)_{measured}$. The maximum measurable optical distance simply depends on the ability of the system to accurately count fringes between two crossing points and the frequency stability of the light sources.

As in our earlier interferometry experiments based on harmonically related light sources, the appropriately chosen pair of light sources and the method of extracting the difference phase allow us to eliminate completely the effect of jitter in the interferometer,[7] which would otherwise make high-precision optical distance measurement impossible. The elimination of jitter also allows us to compare scans performed at different times.

To demonstrate the capability of this technique, we use the system to probe the optical distance between the top and bottom surfaces of a fused-quartz coverslip (physical thickness, $L = 237 \pm 3$ μm). In this case there is a π phase shift associated with reflection from the first interface, which marks a positive refractive-index transition. Hence, there is a $\exp(-i\pi)$ term associated with the factors $R_{LC,1}$ and $R_{cw,1}$ in Eqs. (1) and (2). This results in a correction factor of 1/2 on S_{fringe} and S_{phase}. Figure 2 shows the results of typical scans at the LC wavelengths of 773.0 and 777.0 nm. The results of four scans are summarized in Table 1. The repeatability of the experimental data indicates that the light source is sufficiently stable in frequency.

As can be clearly seen, the experimental data yield an optical absolute distance measurement with subnanometer precision. We note that the optical distance found is associated with the low-coherence light source; the cw heterodyne signal serves as an optical ruler. If L of the quartz coverslip is known precisely, $n_{775\,\mathrm{nm}}$ for quartz at the wavelength 775.0 nm can be found to a very high degree of accuracy from $(n_{775\,\mathrm{nm}}L)_{measured}$.

Alternatively, without knowing the exact value of L, we can find the relative dispersion profile of a sample very accurately (relative to the refractive index of the sample at a given wavelength). Our experimental results predict that a precision of 7 significant figures can be achieved with a 1-mm-thick sample. This compares well with previously developed methods.[8]

As a proof of principle, we change the light sources of the system to a low-coherence superluminescent diode at 1550.0 nm and a cw Ti:sapphire laser at 775.0 nm. By adjusting the operating current through the superluminescent diode, we can change its center wavelength by ~2 nm; this is adequate to achieve phase crossing. Using this system, we can measure the optical distance at 1550.0 nm. Taking the ratio of the result of this measurement with the previous measurement, we can determine $n_{775\,\mathrm{nm}}/n_{1550\,\mathrm{nm}}$ for quartz. The index ratios found are for harmonically related wavelengths; this is simply due to the sources that we have available. Index ratios for other wavelengths can be measured with other appropriate choices of light sources. For comparison, we also present the corresponding data for glass and acrylic plastic (Table 2).

To our knowledge, this technique for optical depth ranging is the first to demonstrate the ability to measure arbitrarily large optical distances with subnanometer precision. We believe that this novel phase-crossing technique for overcoming 2π ambiguity will be of significant use in applications such as high-precision depth ranging and high-precision refractive-index determination of thin-film solid-state materials.

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