Cavity ring-down technique and its application to the measurement of ultraslow velocities

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We have developed a new ring-down technique that does not require a shutter to turn a probe laser on and off. With a rapid cavity scan we can measure a simple exponential cavity decay from which a cavity finesse can be found. When the cavity is scanned slowly, the cavity decay exhibits an amplitude modulation, and an analytic expression is derived for this modulation. With this new technique we measured the ultraslow relative velocity of the mirrors (of the order of micrometers per second) as well as the linewidth (~100 kHz) of the probe laser.

The recent development of an ultra-high-Q optical resonator has made possible many new exciting experiments, such as the experimental realization of the one-atom laser[1] and normal mode splitting for a weakly excited absorbing atom in a cavity.[2] In designing such experiments it is important to know the resonator finesse exactly. One way to measure the finesse is to probe the linewidth of the resonator transmission. Such a measurement is easily done if the resonator linewidth is much broader than the linewidth of the probe laser; if not, a direct measurement of the cavity decay is preferred.

The cavity decay can be measured by use of the decay or ring-down of the field inside the resonator. In the standard ring-down scheme an electromagnetic-field pulse is stored in the empty cavity, and the subsequent decay of the field is monitored in time. The duration of this excitation pulse should be much shorter than the cavity decay time, so that the excitation can be considered to be a delta function. One prepares the excitation field from a cw laser field by turning a shutter, which is often an acousto-optic modulator, on and off. Initially the shutter is opened, and the cavity is permitted to drift slowly toward resonance. At resonance a field builds up quickly in the cavity. When the cavity transmission signal reaches a certain threshold, the acousto-optic modulator is switched off, and the subsequent field decay is measured as a function of time. This technique was used in Ref. 3 to measure a finesse as high as 2×10^6 at 850 nm.

In this Letter we report a much simpler but more powerful ring-down technique that requires neither a shutter nor a trigger circuit to turn the probe laser on and off at the right moment. Instead, we point out that, if the cavity is quickly scanned, it is resonant with the probe for only a brief moment. This short resonance time effectively simulates a delta-function excitation. By controlling the scan speed we can adjust how much field can build up in the cavity before it tunes out of resonance with the laser field. The cavity is scanned repeatedly while the field decay is measured by a photodiode and displayed on an oscilloscope in real time.

The hallmark of this method is that the cavity decay curve shows an amplitude modulation that depends on the scan speed. Note that the cavity configuration in this study (two mirrors and a piezoelectric transducer between them) has been widely used in spectrum analyzers for more than a decade. It may be that the amplitude modulation in the cavity transmission may have been accidentally observed previously but misinterpreted as rf noise or some interference effect, as it was initially so even in our laboratory. Until now the cause for the amplitude modulation has not been well understood. Here we algebraically derive the period of the modulation to be roughly given by

$$T_M \approx \left[\left(\frac{2L_0}{c} \right) \left(\frac{\lambda}{v} \right) \right]^{1/2}. \tag{1}$$

The origin of this modulation is the interference between the probe laser and the intracavity field. Since one of the mirrors is moving, the intracavity field, which built up when the cavity was resonant with the probe field, continuously shifts in frequency owing to the Doppler effect while its amplitude decays. Although the probe laser is no longer resonant with the cavity in a steady-state sense, it can still interact with the cavity field because of the finite transmittance of the mirror that it is incident upon. Since the probe and intracavity field have slightly different frequencies, a beating ensues.

To derive an algebraic expression for the decay curve, we consider a plane-wave laser field that is incident upon a Fabry–Perot resonator. For simplicity, consider the laser to be monochromatic and assume that the distance between mirrors changes linearly with time:

$$L(\tau) = L_0 + v\tau. \tag{2}$$

The velocity v is assumed to be so small that, for the time interval that we consider, $v\tau/L << 1$ or $L(\tau) \approx L_0$. Both mirrors are assumed to have reflection and transmission coefficients r and t, respectively. We

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then obtain the electric field inside the resonator at any instant by summing all the wave components that have undergone multiple reflections:

$$E_{\mathrm{in}}(\tau) = \sum_{n=0}^{\infty} t r^{2n} E_0 \exp\left\{ ik\left[z + 2\sum_{m=1}^{n} L(\tau_m) \right] - i\omega\tau \right\}, \tag{3}$$

where $\tau_m \equiv \tau - (2m-1)L_0/c$. The summation in the exponent can be simplified:

$$\begin{aligned}\sum_{m=1}^{n} L(\tau_m) &= \sum_{m=1}^{n} (L_0 + v\tau_m) \\ &= \sum_{m=1}^{n} [L_0 + v\tau - v(2m-1)L_0 v/c] \\ &= n\left[\left(1 - n\frac{v}{c}\right) L_0 + v\tau \right].\end{aligned} \tag{4}$$

The field inside is thus

$$\begin{aligned}E_{\mathrm{in}}(\tau) &= E_0 \exp[i(kz - \omega\tau)] \\ &\quad \times \sum_{n=0}^{\infty} r^{2n} \exp\left\{ i2nk\left[\left(1 - n\frac{v}{c}\right) L_0 + v\tau \right] \right\}.\end{aligned} \tag{5}$$

We assume that the cavity becomes resonant with the incident field at $\tau = 0$, so that

$$kL_0 = N\pi, \qquad N \text{ an integer}. \tag{6}$$

We also assume that the round-trip time $2L/v$ is much smaller than the cavity decay time, so that an arbitrary time τ can be expressed as $\tau = (2L_0/c)l$, where l is an integer. The phase factor in Eq. (4) then becomes

$$\begin{aligned}2nk&\left[\left(1 - n\frac{v}{c}\right) L_0 + v\tau \right] \\ &= 2nk\left[\left(1 - n\frac{v}{c}\right) L_0 + v\frac{2L_0}{c} l \right] \\ &= 2nN\pi + kv\left(\frac{2L_0}{c}\right) n(2l - n).\end{aligned} \tag{7}$$

Then the intensity is

$$I_{\mathrm{in}}(\tau) \propto \left| \sum_{n=0}^{\infty} r^{2n} \exp\left[ikv\left(\frac{2L_0}{c}\right) n(2l - n) \right] \right|^2. \tag{8}$$

The phase factor in the exponent can be written as a quadratic function of n, which is stationary when $n = l$. If the cavity scan speed is fast enough to make the phase factor much larger than unity at $n = l$, only the terms with n near l can contribute constructively to the summation. In this case, because the r^{2n} factor is a slowly varying function of n, the factor can be taken out of the summation. The intensity then decays exponentially for large l:

$$\begin{aligned}I_{\mathrm{in}}(\tau) &\propto |r|^{4l} \left| \sum_{n=0}^{\infty} \exp\left[ikv\left(\frac{2L_0}{c}\right) n(2l - n) \right] \right|^2 \\ &\propto R^{2l} = \exp(\ln R^{2l}) = \exp\{2l \ln[1 - (1 - R)]\} \\ &\approx \exp[-2(1 - R)l] = \exp[-2(1 - R)c\tau/2L] \\ &= \exp(-\tau/T_{\mathrm{cav}}),\end{aligned} \tag{9}$$

where T_{cav} is the cavity decay time, defined as ${T_{\mathrm{cav}}}^{-1} \equiv c(1 - R)/L$, with $(1 - R) \ll 1$. In general, however, the intensity exhibits a modulation on top of the exponential decay. To see the origin of this modulation, we rewrite Eq. (4) as

$$\begin{aligned}I_{\mathrm{in}}(\tau) &\propto \left| \sum_{n=0}^{\infty} r^{2n} \exp\left\{ ikv\left(\frac{2L_0}{c}\right) [l^2 - (n - l)^2] \right\} \right|^2 \\ &= \left| \sum_{n'=-l}^{\infty} r^{2(n'+l)} \exp\left\{ ikv\left(\frac{2L_0}{c}\right) [l^2 - n'^2] \right\} \right|^2 \\ &= \left| r^{2l} \exp\left[ikv\left(\frac{2L_0}{c}\right) l^2 \right] \right|^2 \\ &\quad \times \left| \sum_{n'=-1}^{\infty} r^{2n'} \exp\left[-ikv\left(\frac{2L_0}{c}\right) n'^2 \right] \right|^2 \\ &= R^{2l} \left| \sum_{n'=-1}^{\infty} r^{2n'} \exp\left[-ikv\left(\frac{2L_0}{c}\right) n'^2 \right] \right|^2 \\ &= R^{2l} \left| \sum_{n''=1}^{l} r^{-2n''} \exp\left[-ikv\left(\frac{2L_0}{c}\right) n''^2 \right] \right. \\ &\quad \left. + \sum_{n'=0}^{\infty} r^{2n'} \exp\left[-ikv\left(\frac{2L_0}{c}\right) n'^2 \right] \right|^2,\end{aligned} \tag{10}$$

where the second term is just a constant corresponding to the field amplitude at $\tau = 0$ ($l \approx 0$) and the first term corresponds to the field amplitude components that have been introduced into the cavity since $\tau = 0$. It is these field amplitudes that cause the total field to exhibit modulation, which occurs because of the sinusoidal nature of the exponential function in the first term. The summation oscillates as a function of l. When l corresponds to $\sim 2\pi m$ of the phase factor of the exponential function, the mth minimum of the decay curve occurs (a numerical simulation shows that the first minimum occurs at 1.8π):

$$kv\left(\frac{2L_0}{c}\right) {l_m}^2 \approx 2\pi m,$$

or

$$\tau_m = \frac{2L_0}{c} l_m \approx \left[m\left(\frac{2L_0}{c}\right)\left(\frac{\lambda}{v}\right) \right]^{1/2}. \tag{11}$$

Therefore the time interval between the first and second minima is

$$T_{12} \equiv \tau_2 - \tau_1 \approx \left(\sqrt{2} - 1\right) \left[\left(\frac{2L_0}{c}\right)\left(\frac{\lambda}{v}\right) \right]^{1/2}. \tag{12}$$

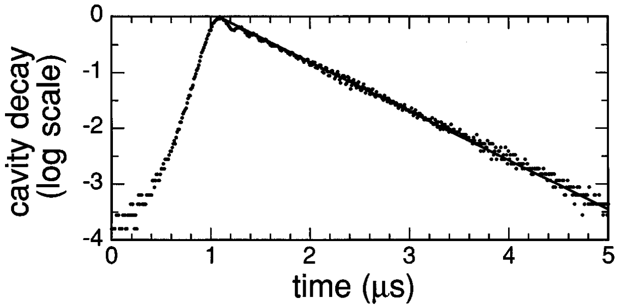

Fig. 1. Typical cavity decay curve obtained with the ring-down technique. The solid curve is an exponential fit to the data.

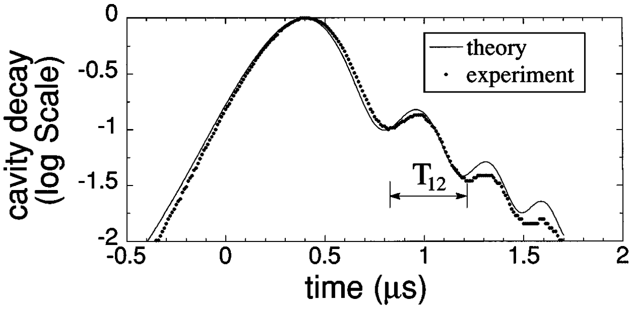

Fig. 2. Cavity decay curve with a slow scan speed exhibiting an amplitude modulation. The period T_{12}, defined in relation (12), is 0.38 μs, resulting in a mirror velocity of 6.4 μm/s.

Note that the oscillating term in the total electric field will be absent if the probe laser is shut off as soon as the cavity resonance is passed. This was the case treated in Ref. 3.

If the probe field has a finite linewidth, the oscillation becomes less pronounced. The effect of laser linewidth can be included in Eq. (3) as an additional random phase factor $\exp[i\phi(\tau)]$, which can be modeled with a correlation function of Gaussian white noise[4]:

$$\{\phi(\tau)\phi(\tau')\}_{\mathrm{ens}} = 2\Gamma_L \delta(\tau - \tau'), \qquad (13)$$

where $\{\}_{\mathrm{ens}}$ denotes an ensemble average and Γ_L is the FWHM laser linewidth.

We measured the cavity decay for various scan speeds and analyzed the results, using the above model. Throughout the experiment the wavelength of the probe laser was 791 nm. A 300-MHz digital oscilloscope (Lecroy 9310M) was used to capture a decay curve in a single scan. The mirror spacing, which was ~1 mm, could be varied by a piezoelectric transducer, which was driven by a voltage ramp from a Tedtronix 555 oscilloscope. A 185-Ω terminator was used at the input of the digital oscilloscope so that the input RC time of the scope was ~30 ns, which was much smaller than a typical cavity decay time of 1 μs. When the cavity was scanned rapidly, a decay curve like the one shown in Fig. 1 was obtained. The solid curve is an exponential fit by a least-squares fit algorithm, resulting in a decay time of 1.14 μs, or a cavity finesse of 1.03×10^6. For these particular data the scan speed was 12 GHz/ms, which corresponds to a mirror velocity of 32 μm/s. According to relation (12), the modulation period T_{12} is ~170 ns, which is consistent with the data. If the cavity is scanned more slowly, the modulation becomes more pronounced. The decay curve in Fig. 2 was obtained with a scan speed of 2.4 GHz/ms, corresponding to a velocity of 6.4 μm/s and a T_{12} of 380 ns, which again is consistent with the data within experimental error. The solid curve is a fit based on the model, with the laser linewidth in the model varied to obtain the best fit. We independently measured the laser linewidth, using a narrow atomic transition (barium $^1S_0 \leftrightarrow {}^3P_1$ transition at 791 nm with a 50-kHz FWHM),[5] to obtain a linewidth of 110 kHz FWHM, which again is consistent with the linewidth of 100 kHz used in the best fit. In this way an unknown laser linewidth can also be estimated.

In conclusion, we have developed a new ring-down technique that is much simpler but more powerful than the conventional ring-down scheme. The new technique is especially useful in measuring an ultraslow velocity (of the order of micrometers per second) of the moving mirror and also provides information on the probe laser linewidth.

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